A Procedure for Estimating Yield Loss from Nutrient Rate Reductions

T. Scott Murrell

Description of the Problem

A common question asked in unfavorable economic times is, "How much yield loss can I expect if I cut my nutrient rates?" While the question appears simple, answering it is not. Generally, when farmers ask this question, they have too little money available to afford the entire quantity of nutrients being recommended. When governments and industries ask this question, they generally have insufficient supplies of nutrients to meet all the needs of the crops in their regions.

Whether or not measurable yield losses will occur when rates are reduced depends primarily on the quantities of nutrients already available in the soil. The higher the quantity of available soil nutrients, the less responsive crops become to nutrient additions. When no fertilizer is applied, soils that have greater quantities of nutrients will produce yields higher than those with lower levels. This general principle operates regardless of the nutrient.

Another consideration is the philosophy used by the adviser making the nutrient recommendation. Using rates that maximize economic returns to nutrients in one season is not always the objective. There may be other factors, such as addressing the uncertainty in characterizing the nutrient supply of the soil, that are more important. Farmers that want to ensure nutrients are non-limiting often apply rates that exceed those necessary for maximizing short-term economic returns. In such cases, cutbacks in application rates may cause no measurable reductions in yield.

A fundamental problem in estimating yield loss is that the yield response to nutrients as well as the soil nutrient supply are often poorly characterized. Just how much an area of the field will yield in a particular year if left unfertilized and how much yield is gained from a nutrient addition are usually not known.

Assumptions for Estimating Yield Loss

In order to come up with an analytical solution to estimate yield loss from rate reductions, we had to make several assumptions that allowed us to properly confine the problem. These assumptions are listed below:

<u>Assumption 1: the recommended rate of the nutrient is an economically optimum one for the upcoming season or crop and ignores residual effects</u>. For phosphorus and potassium, this assumption best fits recommendations that follow the sufficiency philosophy. For nitrogen, this assumption best fits recommendations that do not have the objective of erring on the side of a more liberal application used

to minimize the risk of cutting the supply too short. This assumption implies that yield will be increased upon addition of the nutrient, up to some point of a diminishing return.

Assumption 2: the yield expected by the farmer is the yield corresponding to the economically optimum rate. This assumes that the nutrient in question is the only one that is affecting yield. Additionally, it assumes that the yield is consistent with known performance for the field area.

Assumption 3: relative yields in soil test calibration data provide a reasonable estimate of the yield attained with no added phosphorus or potassium. These percentages are long-term averages and may not accurately reflect the yields attained in any given year and location.

Assumption 4: average relative yields corresponding to no applied nitrogen, published in regional databases of nitrogen response studies, are a reasonable estimate of such yields. These percentages are averaged over many different sites and years and are not sensitive to the specific conditions encountered locally.

Assumption 5: when yield without a nutrient addition is not known and cannot reasonably be estimated, a conservative estimate may be made from the yield response needed to recover the costs associated with the nutrient recommendation. Taking the economic break even yield response and subtracting it from the expected yield in assumption 2) provides a conservative estimate of the yield without the nutrient added. Normally, nutrient recommendations that strive to optimize short-term economic returns produce yield responses much greater than this estimate, making it conservative.

Assumption 6: recommendations for maximizing economic returns were developed using quadratic or quadratic-plateau functions to model crop response to nutrient additions. These equations have been used widely because of their simplicity and their sensitivity to crop and nutrient price.

Mathematical Approach Used to Estimate Yield Loss

Based on assumption 6), we choose the quadratic function to predict yield response to the recommended rate. This function has the form:

$$y = a + bx + cx^2 \quad [1]$$

where y is crop yield, x is nutrient rate, a is the intercept or yield without added nutrient, b is the component of linear slope of the quadratic function, and c is the coefficient of curvature (Figure 1). In this calculation, we limit ourselves to responses that are positive, according to assumption 1). Also according to this assumption, the recommended rate is equated to the economically optimum rate (x_{eor}):

$$x_{eor} = \text{recommended rate} \quad [2]$$

According to assumption 2), the yield associated with this rate is the yield expected by the farmer:

$$y_{eor} = \text{expected yield} \quad [3]$$

Because the response is assumed to be positive, the yield corresponding to the economically optimum rate (y_{eor}) is necessarily higher than the yield without a nutrient addition (a), stated as

$$a < y_{eor} \quad [4]$$

This restriction means that the component of linear slope (b) must be positive:

$$b > 0 \quad [5]$$

A final implication is that the coefficient of curvature (c) must be convex (negative) or zero. If it is convex, the economic optimum rate can be calculated using diminishing returns. If it is zero, a break-even rate can be calculated where the yield response equals the nutrient cost, based on a linear equation involving only b. So:

$$c \leq 0 \quad [6]$$

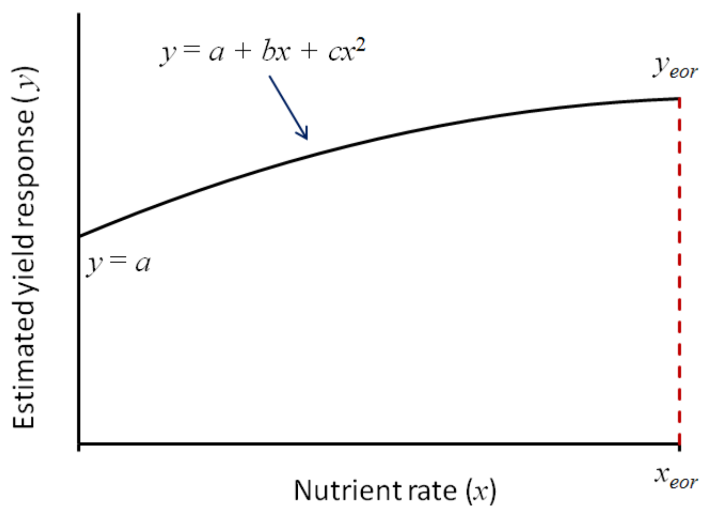

Figure 1. A conceptual model of a quadratic response to a recommended nutrient rate: y is crop yield, a is the yield without any nutrient added, b is linear slope, c is curvature, x is nutrient rate, x_{eor} is the economically optimum nutrient rate, and y_{eor} is the yield produced by the economically optimum rate, assumed to be the recommended rate.

Estimating the value of a

The yield attained without the addition of a given nutrient is not known without experimental testing. In a production field, this yield can be estimated in different ways.

For phosphorus and potassium, soil test calibration data can provide at least a long-term estimate of the percent of yield attained if no fertilizer is added. Calibration data provide relationships between soil test level and relative yield, the latter expressed as percent of the yield attainable when the nutrient is in sufficient supply (Tables 1 and 2). For instance, the Illinois calibration data set indicates that, on average, maize will yield 55% of what is possible if left unfertilized and grown on a soil with a soil test level of 5 ppm Bray P1. Such a percentage, when multiplied by attainable yield, provides a quantitative estimate of the yield expected when no fertilizer is applied.

Table 1. Soil test phosphorus calibration examples (Bray P1 except where noted).

Soil test level (ppm)	Average Relative Yield (%)									
	maize Iowa	maize Mis-souri	maize Illi-nois	maize On-tario*	soy-bean Illi-nois	soy-bean On-tario*	Spring wheat S. Da-kota	Spring wheat N. Great Plains*	Winter wheat Kansas (Bray)	Winter wheat Kansas (Olsen)+
2.5	66.5	31.0	42.0	82	42.0	75	75.2	61.2	35.0	41.0
5.0	77.3	40.0	54.8	86	54.8	87	79.5	78.0	56.4	68.0
7.5	86.7	48.5	69.3	89	69.3	94	83.1	85.9	73.6	82.1
10.0	91.3	58.0	81.3	92	81.3	97	86.0	90.4	82.1	89.9
12.5	94.1	66.5	90.2	93	90.2	98	88.6	93.3	87.9	93.9
15.0	95.9	75.3	94.7	95	94.7	99	91.0	95.4	92.3	97.0
17.5	97.1	84.5	97.3	96	97.3	100	93.1	97.0	95.0	98.5
20.0	98.0	90.0	98.0	97	98.0		94.8	98.2	97.1	99.9
22.5	98.7	93.5	98.6	98	98.6		96.4	99.1	98.2	100.0
25.0	99.3	96.0	99.1	98	99.1		97.8	99.9	99.3	
27.5	99.6	98.0	99.5	99	99.5		98.8	100.0	100.0	
30.0	99.8	99.3	99.8	99	99.8		99.6			
32.5	99.9	99.9	100.0	99	100.0		99.9			
35.0	100.0	100.0		99			100.0			

*Olsen P, +Calculated from Bray P1 assuming Olsen P = 0.75 Bray P1.
Data: Potash & Phosphate Institute, PKMAN: A tool for personalizing P and K management. Version 1.0. Potash & Phosphate Institute, Norcross, GA.

Table 2. Soil test potassium calibration examples (ammonium acetate extractable potassium).

Soil test level (ppm)	Average Relative yield (%)				
	maize Missouri	maize Illinois	maize Ontario	soybean Illinois	soybean Ontario
60	62.0	52.5	90	59.5	87
70	69.8	66.0	92	66.0	90
80	76.3	74.5	94	73.5	92
90	82.0	82.0	95	79.6	94
100	86.8	87.2	96	85.2	95
110	91.0	91.9	97	90.2	96
120	96.0	95.0	98	94.6	97
130	97.0	97.1	98	97.1	97
140	98.3	98.4	99	98.4	98
150	99.6	99.3	99	99.3	98
160	100.0	99.9	99	99.9	99
170		100.0	99	100.0	99

Data: Potash & Phosphate Institute, PKMAN: A tool for personalizing P and K management. Version 1.0. Potash & Phosphate Institute, Norcross, GA.

For nitrogen, some universities have begun publishing average responses of maize to nitrogen rates for given areas. These newly emerging datasets allow, for the first time, estimates of response to be made to this nutrient. In Table 3 we have assembled the average relative yields for no applied nitrogen, by region, taken from the graphs of relative yield and nitrogen rate provided in the user interface to this regional database of nitrogen response studies (http://extension.agron.iastate.edu/soilfertility/nrate.aspx).

Table 3. Average relative maize yield (% of that attainable with sufficient nitrogen) for no applied nitrogen, averaged over responsive sites only and listed by U.S. state or region within a state (http://extension.agron.iastate.edu/soilfertility/nrate.aspx).

U.S. State/region	Average relative yield of maize with no nitrogen after a previous crop of either: maize	soybean
	------------------------ (%)	-------------------
Averaged over responsive sites only		
Iowa	42.5	69.5
Illinois north	51.6	68.7
Illinois central	48.7	57.9
Illinois south	40.4	55.0
Indiana	--	50.2
Minnesota	55.8	71.7
Wisconsin – very high/high soil yield potential	65.5	74.4
Wisconsin – medium/low soil yield potential	59.8	--
Wisconsin – irrigated sands	34.9	34.9
Averaged over responsive and non-responsive sites		
Iowa	44.7	72.1
Illinois north	51.6	68.7
Illinois central	48.7	57.9
Illinois south	40.4	55.0
Indiana	--	50.2
Minnesota	58.5	72.5
Wisconsin – very high/high soil yield potential	70.2	77.1
Wisconsin – medium/low soil yield potential	63.9	--
Wisconsin – irrigated sands	34.9	34.9

In cases when no reasonable estimate for yield without a nutrient application can be made, a conservative estimate is to use the yield response ($y_{eor} - a$) needed to recover the costs of the nutrient product. This minimum required yield response can be written as:

$$y_{eor} - a = \frac{N x_{eor}}{P}$$

where N is the unit price of the nutrient and P is the unit price of the crop. If we define a new variable, R, that represents the ratio of the nutrient price to the crop price (N/P),

$$R = \frac{\text{nutrient price } (N)}{crop\ price\ (P)} \quad [7]$$

the equation becomes

$$y_{eor} - a = Rx_{eor}$$

Rearranging this equation and solving for a provides us with the minimum yield without nutrient that is needed to recover the cost of the nutrient:

$$a = y_{eor} - Rx_{eor} \quad [8]$$

where x_{eor} is the recommended rate (Equation 2) and y_{eor} is the expected yield (Equation 3).

Estimating the values of b and c.

The first step in estimating these coefficients was to express b as a function of c. To create this equation, we used the definition of the economically optimum rate (x_{eor}), which is the rate at which the marginal return is maximized. Marginal return is the change in gross revenue (Pdy) associated with a change in nutrient cost (Ndx). At the maximum marginal return:

$$Pdy = Ndx$$

Rearranging this equation produces

$$\frac{dy}{dx} = \frac{N}{P}$$

Remembering that the crop price ratio (R) equals N/P (Equation 8),

$$\frac{dy}{dx} = R$$

Substituting Equation 1 into this equation yields

$$\frac{d(a + bx + cx^2)}{dx} = R$$

Differentiating Equation 1 with respect to x produces

$$b + 2cx = R$$

Because the rate in this equation is the economically optimum one, we can substitute x with x_{eor}:

$$b + 2cx_{eor} = R$$

Rearranging this equation allows us to express b as a function of c:

$$b = R - 2cx_{eor} \quad [9]$$

Our next step is to use Equation 9 to find an expression we can solve for c. We use the following expression for the yield associated with the economically optimum rate:

$$y_{eor} = a + bx_{eor} + cx_{eor}^2$$

Substituting Equation 9 into the above expression yields

$$\begin{aligned} y_{eor} &= a + (R - 2cx_{eor})x_{eor} + cx_{eor}^2 \\ &= a + Rx_{eor} - 2cx_{eor}^2 + cx_{eor}^2 \\ &= a + Rx_{eor} - cx_{eor}^2 \end{aligned}$$

We next re-arrange this equation and solve for c:

$$c = \frac{Rx_{eor} + a - y_{eor}}{x_{eor}^2} \quad [10]$$

So by knowing the fertilizer to crop price ratio, the recommended rate, the expected yield, and the yield without the nutrient applied, we can find c and, subsequently, b using Equations 9 and 10.

Mathematical Limits for b and c

We stated earlier that when a is not known, it can be conservatively estimated by using Equation 8. We will show that this estimate also satisfies the requirement that c be convex (negative) or zero, according to Equation 6.

By examining the expression for c in Equation 10, we see that c = 0 when

$$Rx_{eor} + a = y_{eor}$$

or

$$a = y_{eor} - Rx_{eor}$$

This is the expression we derived in Equation 8 using the break-even yield response. We also see that c will be negative only when

$$Rx_{eor} + a < y_{eor}$$

or

$$a < y_{eor} - Rx_{eor}$$

Thus to fulfill the requirement in Equation 6, we see that:

$$a \leq y_{eor} - Rx_{eor} \quad [11]$$

The requirement in Equation 11 results in a minimum value for b. This minimum value arises from Equation 8. Using that equation, we see that b is minimized when c = 0:

$$b = R, \text{ when } c = 0 \quad [12]$$

When c < 0, b will be greater than R.

Calculating Yield Loss

Once we have estimates for a, b, and c, we can calculate the expected yield for any rate between 0 and x_{eor} using Equation 1. The difference between the expected yield resulting from a reduced rate and y_{eor} is the expected yield reduction. This reduction can also be expressed as a percent loss, relative to the expected yield, by dividing the reduction by the expected yield and multiplying by 100%.

Example Calculation Using S.I. Units

Let's say we have been asked to estimate the yield loss when a 100 kg K ha^{-1} recommended rate is reduced by 50 kg K ha^{-1}, resulting in an application of 50 kg K ha^{-1} for a soil testing 70 ppm in ammonium acetate extractable K and the yield without added K is not known. The yield level for the area is typically 9400 kg ha^{-1}. The price for maize is \$0.157 kg^{-1}. The price for K is \$1.06 kg^{-1}.

Step 1. Fill in the known values.

x_{eor} = 100 kg K ha^{-1} (Equation 2)

y_{eor} = 9400 kg ha^{-1} (Equation 3)

R = (\$1.06 kg^{-1})/ (\$0.157 kg^{-1}) = 6.75 kg kg^{-1} (Equation 7)

Step 2. Estimate a.

We can get this from the relative yield for one of the states listed in Table 2. Say we choose the maize calibration data set from Missouri. At 70 ppm, the average relative yield is approximately 70%. Multiplying the decimal equivalent by the expected yield provides an estimate of a:

a = (0.70)(9400 kg ha^{-1}) = 6580 kg ha^{-1}.

If we are not comfortable with this estimate, we can calculate a conservative estimate of a using Equation 8:

a = 9400 kg ha^{-1} – (6.75 kg kg^{-1})(100 kg ha^{-1}) = 8725 kg ha^{-1} (Equation 8)

Step 3. Calculate c.

We calculate c using Equation 10:

$$c = \frac{6.75(100\ \text{kg ha}^{-1}) + 6580\ \text{kg ha}^{-1} - 9400\ \text{kg}^{-1}}{(100\ \text{kg ha}^{-1})^2}$$

$$= -0.2145\ \text{kg}^{-1}\text{ha}$$

Step 4. Calculate b.

Calculating b using Equation 9 yields:

$$b = 6.75 - 2(-0.2145\ \text{kg}^{-1}\ \text{ha})(100\ \text{kg ha}^{-1})$$
$$= 49.65$$

Step 5. Calculate the yield expected at the reduced rate of 50 kg K ha^{-1}, using Equation 1:

$$y = 6580\ \text{kg ha}^{-1} + 49.65(50\ \text{kg ha}^{-1}) - 0.2145\ \text{kg}^{-1}\text{ha}\ (50\ \text{kg ha}^{-1})^2$$
$$= 8526\ \text{kg ha}^{-1}$$

Step 6. Calculate the yield reduction by subtracting the expected yield in Step 5 from y_{eor}:

yield reduction = 9400 kg ha^{-1} – 8526 kg ha^{-1} = 874 kg ha^{-1}

or

$$\frac{874\ \text{kg ha}^{-1}}{9400\ \text{kg ha}^{-1}} \times 100\% = 9.3\%$$

Example Calculation Using U.S. Units

Let's say we have been asked to estimate the yield loss when a 110 lb K_2O acre^{-1} recommended rate is reduced by 60 lb K_2O /acre, resulting in an application of 50 lb K_2O/ acre for a soil testing 70 ppm in ammonium acetate extractable K and the yield without added K is not known. The yield level for the area is typically 200 bu acre^{-1}. The price for maize is $4.00 bu^{-1}. The price for K is $0.40 lb K_2O/acre.

Step 1. Fill in the known values.

x_{eor} = 110 lb K_2O/acre(Equation 2)

y_{eor} = 200 bu/acre (Equation 3)

R = ($0.40/lb K_2O) / ($4.00/bu) = 0.1 bu/lb K_2O (Equation 7)

Step 2. Estimate a.

We can get this from the relative yield for one of the states listed in Table 2. Say we choose the maize calibration data set from Missouri. At 70 ppm, the average relative yield is approximately 70%. Multiplying the decimal equivalent by the expected yield provides an estimate of a:

a = (0.70)(200 bu/acre) = 140 bu/acre.

If we are not comfortable with this estimate, we can calculate a conservative estimate of a using Equation 8:

a = 200 bu/acre – (0.1 bu/lb K_2O)(110 lb K_2O/acre) = 189 bu/acre (Equation 8)

Step 3. Calculate c.

We calculate c using Equation 10:

$$c = \frac{(0.1\ \mathrm{bu/lb\ K_2O})(110\ \mathrm{lb\ K_2O/acre}) + 140\ \mathrm{bu/acre}\ - 200\ \mathrm{bu/acre}}{(110\ \mathrm{lb/acre}\ )^2}$$
$$= -0.00405\ \mathrm{bu\ acre/lb^2}$$

Step 4. Calculate b.

Calculating b using Equation 9 yields:

$$b = 0.1\ \mathrm{bu/lb\ K_2O} - 2\left(-0.00405\ \mathrm{bu\ acre/lb^2}\right)(110\ \mathrm{lb\ K_2O/acre})$$
$$= 0.991\ \mathrm{bu/lb\ K_2O}$$

Step 5. Calculate the yield expected at the reduced rate of 50 kg K ha^{-1}, using Equation 1:

$$y = 140\ \mathrm{bu/acre} + (0.991\ \mathrm{bu/lb\ K_2O})(50\ \mathrm{lb\ K_2O/acre})$$
$$- \left(0.00405\ \mathrm{bu\ acre/lb^2}\right)(50\ \mathrm{lb\ K_2O/acre})^2$$
$$= 180\ \mathrm{bu/acre}$$

Step 6. Calculate the yield reduction by subtracting the expected yield in Step 5 from y_{eor}:

yield reduction = 200 bu/acre – 180 bu/acre = 20 bu/acre

or

$$\frac{20\ \mathrm{bu/acre}}{200\ \mathrm{bu/acre}} \times 100\% = 10\%$$

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IPNI, 3500 Parkway Lane, Suite 550, Norcross, GA 30092 USA | T 770.447.0335 | F 770.448.0439