Site-Specific Management Guidelines

P.E. Gaspar, C.G. Carlson, and D.E. Clay

A 'Cookbook' Approach for Determining the 'Point of Maximum Economic Return'

Summary

Many agronomists and producers have been conducting on-farm experiments that are designed to determine the impact of different fertilizer rates or plant populations on crop yields. These data are usually analyzed by plotting the input (fertilizer or population rate) vs. output (yield). The point of maximum yield may be picked directly off the plot. To make the results of these experiments more useful, the point of maximum economic return should be calculated. The point of optimum economic return is determined by:

1. Conducting a yield response experiment;

2. Converting the yield response data to a functional relationship, $output_{corn\ yield} = f(input\ levels)$;

3. Knowing or estimating the costs of your inputs and outputs; and

4. Using calculus to determine where the change in the value of the input equals the change in the value of the output.

The goal of this Guideline is to provide an easy-to-follow approach for calculating the point of maximum economic return.

Agronomists routinely conduct experiments that measure the impact of different fertilizer rates or plant population levels on yield (**Figure 1**). These experiments typically contain five to seven different rates of the input in question (fertilizer rates or population levels) with each rate replicated four times at a site. Details on how to design, set up, and statistically analyze on-farm experiments is available in Marciak (2000) and Wittig and Wicks (1999). The first step in processing this data is to plot the input (x axis) vs. yield (y axis) (**Figure 1**). Many different
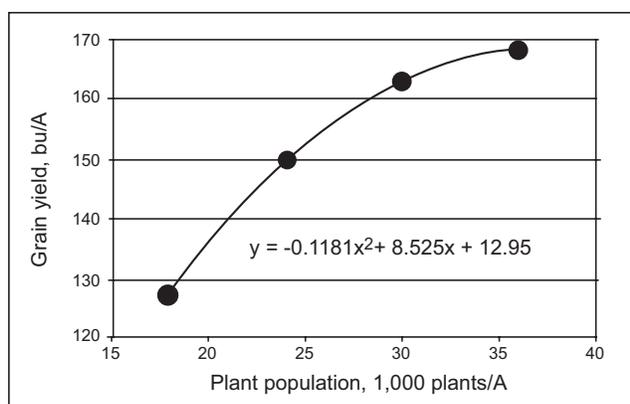
relationships between inputs and yields are possible (decreasing, flat, and increasing). A decreasing relationship (decreasing yield and increasing inputs) can occur when increasing one resource reduces the availability of a different resource. For example, under water stress conditions, increasing the N rate can promote early season water use, which in turn can reduce the amount of water available during grain filling. In this case, yield (y) may decrease with increasing fertilizer (x). If the line is flat, then increasing inputs does not impact yield. In the increasing relationship, the yield may increase until it eventually reaches a plateau (**Figure 1**). This means that there is a positive relationship between input (x) and yield (y). A visual evaluation of the data in **Figure 1** indicates that the highest yield in the experiment was observed at 35,000 plants/A. However, this population level may not be the most profitable. To determine this point, the point of maximum economic return, additional processing is required. A complete discussion of the economic analysis used in these calculations (partial budgeting) is beyond the scope of this Guideline and is available in Dillon and McConnell (1997).

The complete development of the point of maximum economic return analysis is found in the appendix. The analysis is based on the assumption that a short segment of a yield response curve can be accurately described by the 2nd order polynomial ($y = a + bx + cx^2$). In this equation, y

$$y = -0.1181x^2 + 8.525x + 12.95$$

Figure 1. A yield response curve, relating plant population to grain yield. The figure is based on data provided in Gaspar et al. (1999).

is yield and x is a given level of inputs. There may be some error associated with this assumption. A different form of equation (logistic, exponential, logarithmic, etc.) can result in a slightly different result. This equation was selected to simplify the calculations.

Calculations

Determining Points of Maximum Economic Return:

1) **Input data.** Load data from **Table 1** into a spread-sheet such as Microsoft Excel®. Use the same cells as shown in **Table 1**. Your Excel spreadsheet should have columns listed along the top of the page designated by letters (A, B, etc) and rows designated by numbers along the left hand side. On all the tables shown in this guideline, you will not be entering any of the letters for the columns or rows. These are provided for you on the tables as references. For example, when we say the number in cell A2 is 18, we will use the letters at the top and numbers along the left hand side to show the cell in the table. The data are graphically shown in **Figure 1**.

Table 1. Yield response data put into the spreadsheet.

	A	B
1	plant pop	yield
2	18	128
3	24	150
4	30	162
5	36	167

Column headings using letters and row heading using numbers. You will not enter this data on your spreadsheet.

2) **Highlight your data set**. (Highlight means to darken and thus activate the use of a cell or group of cells). To highlight cells A2 to B5, move the mouse cursor to cell A2 and click the left mouse button (cell A2 will be highlighted). Now move the mouse to cell B5, hold the shift down, and click the left mouse button. A2 to B5 now will be highlighted. There are a number of other ways to do this same thing. The end result will be the same regardless of method.

3) **Build Graph**. Click on **Insert, Chart.** A **Chart Wizard** window will appear. On this wizard, select **XY(Scatter),** on the **Chart sub-type** in this window**, select the picture of the top graph that just shows the points,** then select **Finish** [an XY(Scatter) chart will appear on your spreadsheet].

4) **Develop Equation**. Move your cursor to the last point on the chart. A display should come up indicating that this is series 1 point "36"(36,167). With this showing, right click, and a new window appears and you should left click on "**Add Trendline**". In the "Add Trendline" window, select "**Polynomial 2nd**". Now left click on **Options** and check **Display equation on chart**, followed by a left click on **OK.** Your chart will look similar to our **Figure 1**. However, the scale may be different. Your X axis (plant population) may start at 0, whereas we have set our axis to start at 15. Your Y axis (grain yield) may start at 0, whereas we have set ours to start at 120. You can change the scale of the axis by right clicking with your mouse on the axis. A message box will appear, select **Format Axis**. A new

window appears, select the **Scale** tab in this window. On the **Scale** window, set the minimum for the X axis = 15 or if the Y axis was chosen, set the minimum = 120. Remove a check mark in the box in front of minimum in both cases and click **OK**. Now your chart should appear like our **Figure 1**.

5) **Insert Yield Response Equation.** You could transcribe the equation from the chart, but the charts equations may not show enough significant digits. To improve accuracy, **highlight the three cells, A8, B8, and C8**, now type **=linest(B2:B5,A2:A5^{1,2}),** now hold down the **Ctrl, Shift and Enter** buttons simultaneously. This inserts the values for the equation into A8, B8 and C8. Make certain to use the {} brackets on the left and right side of 1,2.

6) If you hit the _Enter_ button after entering the cell equation, **=linest(B2:B5,A2:A5^{1,2}),** this will _not_ give desired results! You will only have 1 number displayed in cell A8. Results should be as seen in cells A8, B8, and C8 in **Table 2**. These are the same constants created by the 2nd order polynomial of **Figure 1** and the coefficients for 2nd order polynomial equation [1] in the appendix.

Table 2. Results in cells A8, B8 and C8 after typing the =linest(B2:B5,A2:A5^{1,2}) equation.

	A	B	C	D	E
7	c	b	a		
8	-0.118055	8.525	12.95		

Values from the polynomial equation shown in blue

7) **Insert Maximum Economic Return Equation**. To develop the equations in cells B10 through E12 in **Table 3**, we will be using equation [11] from the appendix. In cells B9 to E9 type in the range of **expected commodity prices** (B9 refers to corn at $1.5/bu and C9 refers to corn at $2.00/bu). Type in cells A10 to A12 the **cost of the input** (A10 refers to seed corn costing $.75/1,000 seeds or $60/bag and A11 refers to seed corn costing $1.00/1,000 seeds or $80.00/bag). Type into cell B10 the **equation** =($A10/B$9-B8)/(2*A8) from **Table 4**. Note that this is equation [12] from the appendix. Be sure to use the equation exactly as is displayed in **Table 4**. The $ is a control character that holds a column or row constant and is needed to assure that copying down and to the right results in the correct equation.

8) Now, we want to copy the equation from B10 to cells B10 to E12**.** Hold the left button of your mouse to **highlight** cells B10 to B12. Select **Edit** from the menu at the top of the window**,** then select **Fill, Down.** We want to insert the **Maximum Economic Return Equation** into cells C10 through E:12. Hold the left button of your mouse to **highlight** cells B10 to E12. Select **Edit** from the menu at the top of the window**,** then select **Fill, Right.**

9) The values in cells B10 to E12 in your table may have a large number of decimal places. To fix or change the number of decimal places, **highlight** cells B10 to E12 using your mouse; select **Format** from the menu, **Cells, Number tab, Number**. On this window, a box should appear where you can enter the number of decimal places. In this box enter 1 and hit 'Enter' or the 'OK' button at the bottom of this

window. Once you have completed these steps, your table should look like **Table 5**, without the different colors.

Table 3. Expected commodity prices entered in cells B9 through E9 (yellow); cost of inputs entered in cells A10 through A12 (green).

	A	B		E
7	c	b	a	
8	0.118055	8.525	12.95	
9	seed\corn	1.5	2	2.5	3
10	0.75			
11	1			
12	1.25			

- Values from the polynomial equation shown in blue
- Expected commodity price ($/bu) shown in yellow
- Cost of inputs shown in green

Table 4. The equations that need to be entered into the different cells. (Columns C, D, and E left off because of space.) At any time when you are entering the data, you may use the commands tools, options, view, window options, turning on or off formulas, to move back and forth between viewing the spreadsheet's equations and viewing the spreadsheet's results.

	A	B
7	c	B
8	=LINEST(B2:B5,A2:A5^{1,2})	=LINEST(B2:B5,A2:A5^{1,2})
9	seed\corn	1.5
10	0.75	=($A10/B9-$B$8)/(2*$A$8)
11	1	=($A11/B9-$B$8)/(2*$A$8)
12	1.25	=($A12/B9-$B$8)/(2*$A$8)

Table 5. Most profitable plant populations (*1,000) are shown in pink cells for a given yield response (equation coefficients shown in blue cells) and different expected commodity prices (shown in yellow cells) and different seed input costs (shown in green cells).

Values from the polynomial equation shown in blue

	A	B	C	D	E
7	c	b	a		
8	-0.118055	8.525	12.95		
9	seed\corn	1.5	2	2.5	3
10	0.75	34.0	34.5	34.8	35.0
11	1		34.0	34.4	34.7
12	1.25	32.6	33.5	34.0	34.3

- Expected commodity price ($/bu) shown in yellow
- As an example, 34,700 plants/A is the most profitable plant population, given the yield response curve and corn valued at $3.00/bu with seed costing $1.00/1,000 seeds or $80.00/bag.
- Cost of inputs shown in green

Discussion

Using the Calculated Point of Economic Return as an Analysis Tool

In an agronomic production system there are always dynamic (changing) variables that are difficult to estimate. Among them are the price per bushel that the commodity will be sold for and the price that will be paid for the input (in this case, seed). To account for this uncertainty, it is best to analyze across the range of expected input and output values. In our case, corn prices could reasonably range from $1.50/bu to $3.00/bu (B9:E9 of **Table 3**). Seed could reasonably range in cost from $60.00/bag (which is $0.75/1,000 seeds) to $100.00/bag (which is $1.25/1,000 seeds, A10:A12 of **Table 3**). Listed in cells B10 to E12 of **Table 4** are the solutions to the analysis (equation [11] of

the appendix). This equation [11] is only appropriate if the yield response is explained by using a 2nd order polynomial equation. These results are the points of maximum economic return for the input – output scenarios discussed. For example, in cell B10 the value calculated and shown is 34.0. For the yield response curve (cells A8:C8) that we have accepted, if corn is sold for $1.50/bu (cell B9), and seed is purchased for $0.75/1,000 seeds (cell A10), the most profitable plant population is 34.0 (cell B10) thousand seeds/A or 34,000 seeds/A.

The above analysis only determines the point of maximum economic return. Often it is important to determine the relative profitability of different management options. Partial budgeting or enterprise analysis can be used to determine the relative profit of a specific management decision. In partial budgeting, only the cost and benefit from a single decision are considered (Dillon and McConnell, 1997). In our simple example, the profitability is defined by the following equation:

Profitability $=$ Yield $_{(input\ level)} *$ ($ Corn value) – (Seed costs)   [1]

This equation can be rewritten into the form used by the Excel® spreadsheets. This equation is:

Yield Response $= (-0.1181x^2 + 8.525x + 12.95) * $ CV $-$ (SP/SC) [2]

where x is corn population/level shown in B25 (**Table 6**, below); CV is value of corn/bu (B24, **Table 6** below); SP is seed population in number/1,000 seeds (C25, **Table 6** below) and SC is cost per 1,000 seeds (C24, **Table 6** below). After typing in the data and equation shown in **Table 6**, line 26 should match the data shown in line 26 of **Table 7**. All you need to do is enter the equation as shown in B26 and then highlight B26 through E26, hit Edit, Fill right, and Enter. The other three figures will automatically be calculated and shown.

Table 6. Equations used to calculate profit for a given input level. (Columns D and E left off because of space.) To view these equations as shown, in Excel select: tools, options, view, window options, formulas mode.

	A	B	C
		$/bu	$/1000 seeds
24		2	1
25	plant pop.	26	30
26	99	=(-.1181*B25^2+8.525*B25+12.95)*$B24-B25*$C24	=(-.1181*C25^2+8.525*C25+12.95)*$B24-C25*$C24

Table 7. The profitability as defined by equation 1 of 4 different plant populations (26,000, 30,000, 34,000, and 38,000 plants/A).

- Values of corn
- Cost of seed

	A	B	C	D	E
		$/bu	$/1000 seeds		
24		$2.00	$1.00		
25	plant population	26	30	34	38
26	99	$283.53	$294.82	$298.55	$294.73

- Profit
- Plant population

The potential profit for the specific input levels, as defined by equation 1, are shown in cells B26 through E26. The costs and expected returns used in this scenario can be changed by changing the value in B24 and C24. In an enterprise analysis, only the cost and value from a management decision are considered. If 26,000 seeds/A is the standard practice, then the benefit of increasing the seeding rate can be directly calculated from the data set. Increasing the seeding rate from 26,000 to 34,000 seeds/A increased the potential profit $15.02/A ($298.55 – $283.53). This analysis can be shown graphically in **Figure 2** (we have not provided the steps for making this figure; it is provided to illustrate the above discussion). This analysis only considers the data shown in the yield response curve (**Figure 1**). Increasing the population may also increase crop diseases, insect infestations, and harvest losses. These factors also need to be considered in making a management decision. A more complete description of results from experiments (conducted under a range of environments) relating population to yield is available in Paskiewicz and Butzen (2002).
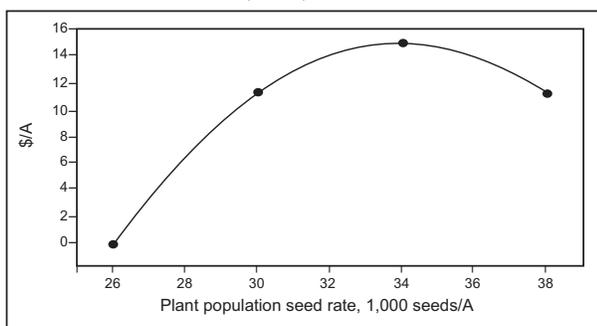

Figure 2. **Estimate of potential profitability associated with different seeding rates. Potential profits were estimated using a partial budgeting approach where the seeding rate of 26,000 seeds/A was defined as initial conditions.**

Conclusions

Any appropriate input–output relationship that is a continuum and can be expressed as a 2nd order polynomial, can be analyzed using this method. In addition to the plant population example of this paper, other good yield response examples are yield as a function of nitrogen, and dairy milk production as a function of alfalfa hay quality (relative feed value). This is a tool that belongs in your thinking toolbox. You can always look in hindsite at a set of yield response data and conclude with confidence what the best recommendation should have been. Use of this tool will increase a crop manager's skills, which in turn will help producers get the most out of their inputs. ■

References

Dillon, J.L. and D.J. McConnell. 1997. Farm management of Asia: A Systems Approach. (FAO Farm System Management Series-13). Food and Agriculture Organization of the United Nations. Available online at http://www.fao.org/docrep/w7365e/w7365e00.htm#Contents (accessed 1/03).

Gaspar, P.E., S. Paszkiewicz, P. Carter, M. McLeod, T. Doerge, and S. Butzen. 1999. Corn hybrid response to plant populations. Pioneer Hi-Bred International Inc. Northern Agronomic Research Summary. p. 29-40.

Marciak, L. 2000. Experimental Design. Available at http://www.agric.gov.ab.ca/crops/fieldexp/experim.html (accessed 1/03).

Paszkiewicz, S. and S. Butzen. 2002. Corn hybrid response to plant population. Pioneer product information. Available at http://www.pioneer.com/canada/pro%5Fservices/mmax/population%5Fresponse.htm. (accessed 2/03).

Wittig, T.A. and Z.W. Wicks II. 1999. On-farm comparisons. SSMG – 18. Clay et al. (ed.) Site-Specific Management Guidelines. Available at http://www.ppi-far.org/ssmg. (accessed 1/03).

Appendix

The mathematics used to develop equations used in the Excel® spreadsheets are shown below.

$$Yield = f(Input) \qquad [1]$$

Which says that yield is a function of input. You could also say that as the value of the input changes, the value of the yield changes. This relationship between yield and inputs can be described by a 2nd order equation.

$$Yield = a + b*Input + c*Input^2 \qquad [2]$$

where, a = 12.95, b = 8.525, and c = -0.1181.

This equation is shown in **Figure 1**. The point of maximum economic return is determined by the equation:

$$\mathbf{d}Yield\$ = \mathbf{d}Input\$ \qquad [3]$$

which says the change in the dollars of output (increased value of the crop) = the change in the dollars of input (seed cost). Note for those who understand calculus: the **d** in the equation indicates the derivative in our example.

$$Yield\$ = Yield * \$/bu\ corn \qquad [4]$$

$$Input\$ = Input * \$/1,000\ seeds\ of\ corn \qquad [5]$$

Using algebra, equation [3] can be rewritten; $\mathbf{d}Yield\$/\mathbf{d}Input\$ = 1$ [6]

substituting [4] and [5] into [6];

$$(\mathbf{d}\{Yield*\$/bu\ corn\})/(\mathbf{d}\{Input*\$/1000\ seeds\ of\ corn\}) = 1 \qquad [7a]$$

or $(\mathbf{d}Yield*\{\$/bu\ corn\})/(\mathbf{d}Input*\{\$/1000\ seeds\ of\ corn\}) = 1$ [7b]

with manipulation [7b] becomes

$$\mathbf{d}Yield/\mathbf{d}Input = \{\$/1,000\ seeds\ of\ corn\}/\{\$/bu\ corn\} \qquad [8]$$

differentiate equation [2] and get

$$\mathbf{d}Yield/\mathbf{d}Input = 2*c*Input + b \qquad [9]$$

then substituting equation [9] into equation [8] yields;

$$2*c*Input + b = \{\$/1,000\ seeds\ of\ corn\}/\{\$/bu\ corn\} \qquad [10]$$

solving [10] for Input, the input point of maximum economic return is equal to: Input $= (\{\$/1,000\ seeds\ of\ corn\}/\{\$/bu\ corn\} - b) / (2*c)$ [11]

And Profit, shown in line 26 of **Table 7**, is calculated using the equation:

Profit $= (a + (b *input) + (c *input^2)) *\$/bu\ corn - input *\$/1,000$ seeds of corn where a = -0.1181, input = B25, b = 8.525, c = 12.95;

$\$/bu$ corn = B24 and $\$/1,000$ seeds of corn = C24 [12]

$= (\$A10/B\$9 - \$B\$8) / 2 *\$A\$8)$

This Site-Specific Management Guideline was prepared by:

Dr. Paul E. Gaspar
Agronomy Research Manager
Pioneer Hi-Bred International, Inc.
19456 State Highway 22
Mankato, MN 56001
Phone: (507) 625-3252 Ext. 224
E-mail: Paul.gaspar@pioneer.com

Dr. C. Gregg Carlson
Professor, Soil Science
South Dakota State University
Brookings, SD 57007
Phone: (605) 688-4761
E-mail: Gregg_Carlson@sdstate.edu

Dr. David E. Clay
Professor, Soil Science
South Dakota State University
Brookings, SD 57007
Phone: (605) 688-5081
E-mail: David_Clay@sdstate.edu