

# The Law of the Maximum

By Arthur Wallace

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*Two different Laws of the Minimum are used to describe how limiting factors relate to crop production. Both came from Germany. One was formulated in 1843 and the other in 1909. The first carries the name of Liebig, who was the pioneer of the concept of mineral based plant nutrition. The second carries the name of the scientist who developed it—Mitscherlich.*

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**THE** Liebig Law of the Minimum says that crops yields are regulated by the factor in greatest limitation, and yields can be increased only by correction of that limiting factor. When that limitation is overcome, yields are then regulated by the next important limiting factor. Further yield increase will then occur only if that factor is corrected. This process is repeated, with step-wise yield increases, until there are no remaining limiting factors.

In a different approach, the Mitscherlich Law of the Minimum states that yields are influenced by all limiting factors simultaneously. The influence of each of the limiting factors is proportional to its degree of limitation. With this law of the minimum the yield for any given mix of conditions is related to the integrated sum of all of the remaining limiting factors. It is possible to mathematically express the degree of each limitation for each factor from laboratory and field tests. From the data, it is then possible to calculate the expected yields as inputs are made to correct the various limitations.

Actually, conditions exist where both laws of the minimum operate, and, more importantly, responses differ for each. Some limiting factors are so severe that no, or relatively little, increase in yields from inputs to correct less limiting inputs is possible unless the severe ones are first corrected. Without correction of these severe limiting factors, other less needed inputs may even cause a yield decrease. The severe limiting factors fit the Liebig law. In contrast, and only when there are no remaining Liebig-limiting factors,

favorable responses can be obtained for any input that corrects a limiting factor of the Mitscherlich type. It really does not matter in which order these limiting factors are corrected as long as they are in keeping with the physiological status of the crop. The order does matter for those limitations of the Liebig type; the most severe ones must be corrected first or response to the less severe ones will be minimal.

The important purpose of soil and plant analyses is to identify limiting factors which can be corrected. Both types of limiting factors can be properly labeled. The more limiting factors that are corrected, the higher the yield will be, provided all those of the Liebig type have been corrected.

## Interactions

Instead of laws of the minimum, we now can have a **Law of the Maximum**. The Law of the Maximum cannot operate if there are any Liebig-type limiting factors present. It has two major characteristics. First, the effect of a given input is progressively magnified as other limiting factors are corrected. The final result is greater than the sum of the effects of the individual inputs because of the way in which they interact. The interaction multiplies the effects of each. Second, yields can be highest or maximum only if there are no remaining limiting factors; the fewer limiting factors that remain, the higher will be the yield. How closely this can be approached, of course, depends

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upon economics. Fortunately, when dealing with Mitscherlich-type factors, those most economical to use can be chosen first.

### Examples

Some examples of how the Law of the Maximum operates are shown.

An application of potassium (K) resulted in an orange yield increase of 81 lb per tree; when applied simultaneously with phosphorus (P), the increase attributable to K was 115 lb per tree; when applied with both P and nitrogen (N), the increase attributable to K was 202 lb per tree (University of California data). The same amount of K was applied in each case. Potassium was almost two and one-half times as valuable when applied with N and P as when applied alone.

Yield response of sugarcane due to K increased progressively from 4.4 tons/A to 5.7 tons/A when another limiting factor was corrected, to 6.6 with a third, to 9.8 with a fourth, to 12.7 with a fifth, to 14.0 with a sixth, and to 16.8 with a seventh (Pakistan Sugar Company data). The yield increase due to the K was multiplied 3.8 times as additional limiting factors were corrected. The value of the other inputs also increased with more K.

Further explanation as to how the various inputs interacted in this sugarcane experiment is indicated by the relative response to each of the seven management inputs into the system. With the control equated to 1.00, the values were 1.30 for a hot water seed treatment to control a fungal disease, 1.15 for a fungicide, 1.50 for urea management, 1.3 for a herbicide, 1.30 for a K treatment, 1.10 for a micronutrient program, and 1.20 for banding of additional nutrients. The final yield was  $1.30 \times 1.15 \times 1.50 \times 1.30 \times 1.10 \times 1.20 = 5.00$  or a five-fold yield increase. The response was according to multiplication, not addition, which would total only 185 percent. This effectively explains the Law of the Maximum.

In another example, P increased the yield of cabbage by 5.7 tons/A, and K increased it by 7.0 tons/A. Both together

increased it by 16.5 tons/A (Cornell University data). The result is greater than the sum of the parts, which is an important characteristic of the Law of the Maximum. The value of an input increases as other limiting factors are corrected. In this case, the increase for K alone was 5.7 tons/A and 10.8 tons/A when used with P.

These examples give new insights to maximum economic yield (MEY). A grower will obtain much more for each input dollar when as many limiting factors as possible are corrected simultaneously, especially those which are inexpensive to correct. The process has been called best management practices (BMPs), but could be called High-Precision Practices.

To maintain an intensively managed production system, it is essential that all limiting factors are identified, and the degree of limitation that each is responsible for be known. This is possible with a combination of laboratory diagnosis, field investigation, integration of available research, and crop experience data.

Some arithmetic with the variables in the sugarcane experiment gives degree of adequacy (we call it Multiple Action Yield Fraction) as 0.77 for seed treatment, 0.87 for fungicide treatment, 0.67 for urea treatment, 0.77 for the weed effect, 0.77 for the K treatment, 0.91 for the micronutrients, and 0.83 for the other nutrients. All these values multiplied together gives 0.20 or 20 percent. The yield for the control was 15 tons/A, approximately 20 percent of the highest yield which was 73 tons/A.

These calculations indicate how potent the effects of high precision can be and how devastating slight departures from the exact needs for crop production can be. For example, if 100 percent were the yield attainable and all factors except one were optimal, the final yield would be whatever that one factor represented whether it be 50 percent, 80 percent, or 90 percent. Two such factors each at 90 percent of limitation would give 81 percent of the yield attainable ( $0.90 \times 0.90 = 0.81$ ). For five such factors the yield would be 59 percent, and for 10 it would be only 35 percent. This is just about where the U.S.

stands in most of its crop production. The maximum corn yield that has been obtained is 370 bu/A; 35 percent of that is 130 bu/A, which is near national average. A farmer may do everything to 90 percent of perfection and yet get only 35 percent of the possible yield.

Improvement requires the pinpoint precision that laboratory diagnosis, computerized programming for decision making and expert consulting can provide. It may be little more costly to get 95 percent of perfection. But for 10 factors, the 35 percent yield would increase to 60 percent of that attainable, which for corn could be 220 bu/A. Some farmers do make it. The principles involved use the Law of the Maximum.

In reaching high yields with high precision, it is emphasized that excesses of input are avoided. Only that needed is used; environmental problems are avoided. Excess inputs may contribute to decreased yields anyway.

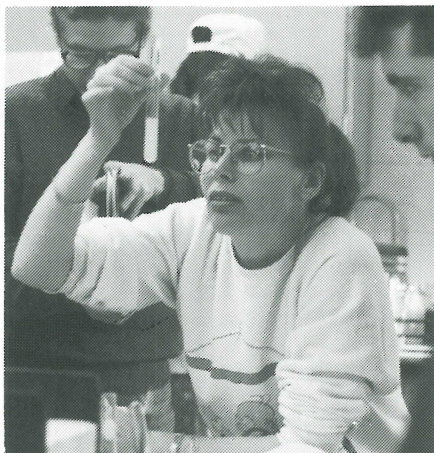
Careful planning is essential for farmers to obtain sufficient precision of inputs to approach the record yields, such as 370 bu/A of corn, or 6.5 tons/A (216 bu/A) of wheat, or 118 bu/A of soybeans. The effects of inputs and their interactions can be mathematically programmed. How closely record yields can be approached will depend upon skillful use of the Law of the Maximum together with use of economic and environmental principles and realities. ■

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